Int. J. Heat Mass Transfer. Vol. 30, No. 7, pp. 1556–1558, 1987
Printed in Great Britain

0017-9310/87 \$3.00+0.00
© 1987 Pergamon Journals Ltd.

Free stream effects on the wave instability of buoyant flows along an isothermal vertical flat plate

S. L. LEE,† T. S. CHEN and B. F. ARMALY

Department of Mechanical and Aerospace Engineering, University of Missouri–Rolla, Rolla, MO 65401, U.S.A.

(*Received* 8 *August* 1986 *and in final form* 28 *January* 1987)

INTRODUCTION

THERE is a lack of literature on wave instability of strong buoyant flows in the presence of a weak forced flow in the free stream. The paper by Carey and Gebhart [1] appears to be the only one that has dealt with such a problem. They solved the standard Orr–Sommerfeld equation with coupling of the temperature disturbance amplitude equation for a buoyant flow with forced free stream along a vertical flat plate under the uniform surface heat flux condition. Numerical results were presented for $Re_x/Gr_x^{*1/4}$ values of -2, 0 and 4 for air ($Pr = 0.733$) and -0.8, 0, and 1.6 for water ($Pr = 6.7$). Based on their results, they concluded that for gases an aiding forced flow reduces the amplification rate as compared to purely natural convection flow (i.e. it has a stabilizing effect on the flow), whereas an opposing forced flow tends to destabilize the flow, and that the effect of an aiding or opposing forced flow is almost negligible in water ($Pr = 6.7$). However, their results also reveal that an aiding forced flow decreases the critical Grashof number for air ($Pr = 0.733$).

The purpose of this investigation is to examine the free stream forced-flow effect on the wave instability of strong buoyant flows along an isothermal vertical flat plate. An efficient and accurate finite difference method proposed by Lee *et al.* [2] is employed to solve the modified Orr–Sommerfeld equations, with a coupling between the amplitude functions for disturbance velocity and temperature, in which the nonparallelism of the mainflow is taken into account. Stability curves are presented and characteristics of disturbance amplification are also examined for $Pr = 0.7$ and 7. These results cover the forced flow parameter, $\xi = [2(Gr_x/Re_x^2)^{1/2}]^{-1}$, in the range of $0 \leqslant \xi \leqslant 0.45$ (or buoyancy parameter, Gr_x/Re_x^2, in the range of $1.235 \leqslant Gr_x/Re_x^2 \leqslant \infty$).

ANALYSIS

For mixed convection flow along an isothermal vertical flat plate, the modified Orr–Sommerfeld equations based on the linear theory and the quasi-parallel flow model (in which the nonparallelism of the mainflow is taken into account) have the following dimensionless form [2]

$$\phi^{iv} + a_1\phi''' + a_2\phi'' + a_3\phi' + a_4\phi + a_5 s' = 0 \quad (1)$$

$$s'' + b_1 s' + b_2 s + b_3\phi' + b_4\phi = 0. \quad (2)$$

The associated boundary conditions are

$$\phi(0) = \phi'(0) = \phi(\infty) = \phi'(\infty) = 0$$
$$s(0) = s(\infty) = 0. \quad (3)$$

In the above equations (1)–(3), the primes stand for derivatives with respect to η and

$$a_1 = -v, \quad a_2 = -2\alpha^2 - i\lambda(\alpha u - \omega),$$
$$a_3 = \alpha^2 v + v''$$
$$a_4 = \alpha^4 + i\alpha\lambda(\alpha^2 u - \omega\alpha + u''), \quad a_5 = 1 \quad (4)$$
$$b_1 = Pr\, a_1, \quad b_2 = -\alpha^2 - i\, Pr\, \lambda(\alpha u - \omega)$$
$$b_3 = -Pr\,\Omega, \quad b_4 = i\alpha\lambda\, Pr\, \theta'.$$

The dimensionless quantities are defined as

$$\phi = \tilde{\phi}/\delta U_c, \quad s = \tilde{s}/T_c, \quad \theta = (T - T_\infty)/T_c$$
$$\alpha = \tilde{\alpha}\delta, \quad u = U/U_c, \quad \omega = \tilde{\omega}\delta/U_c, \quad \eta = y/\delta \quad (5)$$
$$v = V\lambda/U_c, \quad \lambda = 4(Gr_x/4)^{1/4}, \quad \delta = \sqrt{2}x/Gr_x^{1/4}$$
$$\Omega = \lambda\delta\, \partial\theta/\partial x, \quad U_c = 2(\nu/x)Gr_x^{1/2}, \quad T_c = T_w - T_\infty$$

where $\phi(\eta)$ and $s(\eta)$ are, respectively, the dimensionless amplitude functions of the disturbance stream function and temperature. The tilde '~' denotes dimensional quantities of the disturbances. The wave number α is a complex number (i.e. $\alpha = \alpha_r + i\alpha_i$), and the wave frequency ω is a real number. The quantities U_c and T_c denote the characteristic velocity and the characteristic temperature, respectively. It is pointed out that the present analysis is performed only at a certain streamwise location x. Hence, $\delta(x)$, a measure of boundary layer thickness, can be treated as a constant in the dimensionless transformation, equation (5).

For the problem of strong buoyant mixed convection boundary layer flows along an isothermal vertical flat plate, with an aiding forced flow in the free stream under study here, the dimensionless mainflow quantities, u, u'', v, v'', and Ω in equation (5) are expressible as

$$u = f', \quad u'' = f'''$$
$$v = \eta f' - 3f + 2\xi\, \partial f/\partial\xi$$
$$v'' = \eta f''' - f'' + 2\xi\, \partial f''/\partial\xi \quad (6)$$
$$\Omega = -(\eta\theta' + 2\xi\, \partial\theta/\partial\xi)$$

in which the quantities f, f', f'', θ and θ' are obtained from solutions of the following system of equations [3]

$$f''' + 3ff'' - 2f'^2 + \theta = 2\xi(f''\, \partial f/\partial\xi - f'\, \partial f'/\partial\xi) \quad (7)$$

$$\theta'' + 3\, Pr\, f\theta' = 2\, Pr\, \xi(\theta'\, \partial f/\partial\xi - f'\, \partial\theta/\partial\xi) \quad (8)$$

and

$$f(\xi, 0) = f'(\xi, 0) = f'(\xi, \infty) - \xi = 0$$
$$\theta(\xi, 0) - 1 = \theta(\xi, \infty) = 0. \quad (9)$$

Note that when the forced-flow parameter $\xi = 0$, both the mainflow problem (equations (7)–(9)) and the modified Orr–Sommerfeld problem (equations (1)–(3)) reduce, respectively, to those presented in ref. [2] for the case of pure free convection along an isothermal vertical plate. Strictly speaking, the amplitude functions ϕ and s depend on both ξ and η in boundary layer flows, because such a flow is nonparallel (see, e.g. Gaster [4]). However, as a preliminary

† Permanent address: Department of Power Mechanical Engineering, National Tsing-Hua University, Hsinchu, Taiwan, Republic of China.